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The Value of π in Ancient Indian Vedic and Jaina Texts

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Abstract;

India is recognized as one of the most important ancient country where the civilizing process was initiated and developed. The remarkable contribution of Indian scholars to the field of mathematics is well mentioned and explained in ancient Indian Vedic and Jaina philosophical texts. Ancient Indian mathematical contribution with special reference to the value of π is explored in this paper with the help of Vedic and Jaina texts which are in ancient Indian languages. The Vedic and Jaina contribution in the field of mathematics is remarkable and it is of around 500 BC or may be of earlier period. However the various textual evidences lead to the conclusion that Indians knew the value of π .

Key Words: Siddhanta , Sutra, Rigaveda,, Samhita, , Jaina, Vedic, Sulba-Sutra , Gatha

Introduction:

Ancient and medieval Indians were aware that there is a fixed ratio between the circumference and diameter of a circle. This ratio is now known by the Greek letter π. Its value is transcendental and non- terminating. Round the beginning of the Christian era say a century on either side of it, a new class of Indian mathematical literature emerged. Remarkable development of newer mathematical methods with the invention of the decimal system greatly promoted mathematical principles. It is all mentioned in our ancient Indian religious and philosophical manuscripts. Only we have to rediscover it and put it in systematic way. The aim of this paper is to trace the discovery of the value of π in ancient India both in Vedic and Jaina texts.

1. Value of π in the Texts of Vedic period :

We find references of $\pi = \sqrt{10}$, in many ancient texts of Vedic period.

1. Ancient Surya Siddhanta before the period of Aryabhata I and the available Surya Siddhanta (I , 58) .
2. Vishnudharmottariya Paitamaha Siddhanta (III , 6) .
3. Pancasiddhantika of Varaha Mihira of 550 AD (IV , 1) .
4. Brahama Sphuta - Siddhanta of Brahmagupta , 628 AD (XII, 40) .
5. Aryabhattiya Bhashya on Bhashkara I (629 AD) , Delhi Ed. , 1976 , p. 72.
6. Trishatika by Shridhara (Sutra 45) .

The Sutra of Shridhara is also quoted by Shankara Variyara in the commentary Kriyakarmakari Tika on Lilavati of Bhaskaracharya .

In fact the credit of giving for the first time in India the value of π correct to four places of decimal as 3.1416, goes to Aryabhata I (476 AD). His achievement in the 5th century is truly significant in the light of the fact that it was only thirteen centuries later in the year 1761, that Lambert proved that π is irrational and in the year 1882 that Lindeman established that π is transcendental.

Its value is given by Aryabhata I in the following words [1]

"Add 4 to 100 and multiply by 8 and add 62000 this is an approximately circumference of a circle whose diameter is 20000 units. This means the circumference of the circle is 62832 units. Aryabhata I took the value of π approximate as 3.1416.

This gives

$$\pi = \frac{\text{Circumference}}{\text{Diameter}} = \frac{62832}{20000} = 3.1416$$

It is remarkable that Aryabhata I was the first Indian mathematician to have given the value of π which is correct up to the four places of the decimals [2].

It is to be noted that Varaha Mihira , Brahmagupta , Shridhara and Shripati had used the value of π as given by Jaina texts . Even in the Siddhanta Tatva a book on astronomy of 17th century AD written by Kamalakar, the value of $\pi = \sqrt{10}$ was considered . At that time the other subtle (sukshma) or more accurate value was known to mathematicians of India and other countries. The value 3 for π is much older value and was accepted in ancient time. It is also found in the Mahabharata [3] and the Vayu Purana [4], Baudhayana - Sulba Sutra [5], with Sanskrita commentary by Dwarka Nath Yajavan .

2. Value of π by Kerala School of Mathematics:

Madhava , the great pioneer of Kerala School of Mathematics (1340 AD – 1425 AD) had even better approximation [6] to $\pi = 3.1415926536$. The Indian mathematicians had good knowledge of irrational numbers and idea of infinite series. Yuktibhasha of Jyesthadeva (c.1530) contains usual mathematical material and with a proof of Pythagorean Theorem, approximation of circumference of the circle by the perimeters of inscribed regular polygons and most importantly an infinite series for $\frac{\pi}{4}$. The power expansion for $\tan^{-1} x$ is given in this text but formally the mathematicians had no idea about convergence of the series. All these were achieved [7] at least two centuries before Europe ever came to consider these questions.

We know the Gregory's Series ---

If θ lies between $-\frac{\pi}{4}$ and $\frac{\pi}{4}$

$$\text{Then} \quad \theta = \tan\theta - \frac{1}{3}\tan^3\theta + \frac{1}{5}\tan^5\theta - \ldots$$

Now we give a proof of the same result due to Kerala School of Mathematics which is discussed in Yuktibhasha [8] as follows --

$$\text{If } c = 4d - \frac{4d}{3} + \frac{4d}{5} - \ldots$$

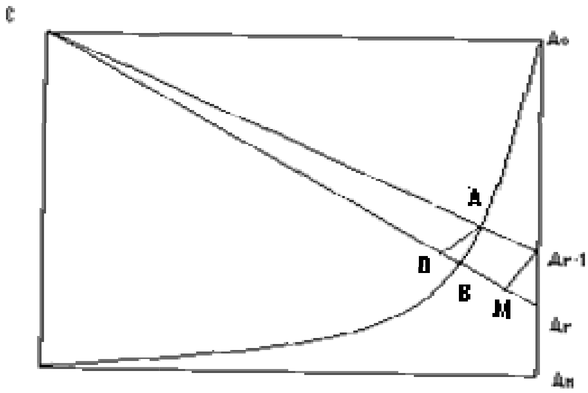
Where c is the circumference and d is the diameter of the circle or equivalently

$$\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \ldots .$$

Consider a quarter of a circle having radius equal to unity with centre C inscribed in a unit square as shown in the figure :

Partition of $A_0 A_n$ in to n equal parts . Let CA_{r-1} , CA_r meet the circle at A and B We draw AD and A_{r-1} M perpendiculars to CA_r from A and A_{r-1} respectively. Consider the two triangles CAD and CA_{r-1} M which are similar. So we can write

$$\frac{AD}{CA} = \frac{A_{r-1} M}{CA_{r-1}} \qquad \text{------ (1)}$$



Geometrical construction used in the proof of the infinite series for π

From the similarity of the triangles M A_{r-1} A_r and CA_0 A_r we get

$$\frac{A_{r-1} M}{A_{r-1} A_r} = \frac{CA_0}{CA_r} \qquad \text{------- (2)}$$

So we get by 1 and 2

$$AD = \frac{CA.\ CA_0\ .A_{r-1}\ A_r}{CA_{r-1}\ .\ CA_r}$$, Here CA and CA_0 are of unit length.

So we have $$AD = \frac{A_{r-1}\ A_r}{CA_{r-1}\ .\ CA_r}$$

Now if we take n sufficiently large , the segment AD tends to the arc AB and CA_{r-1} and CA_r become approximately equal so that we have Arc AB is the approximately equal to side AD.

$$\text{Arc AB} \sim \frac{1/n}{CA_{r-1}^2} = \frac{1/n}{1+(\frac{r-1}{n})^2}$$

Note $CA^2_{r-1} = CA^2_0 + A_0 A^2_{r-1} = 1 + A_0 A^2_{r-1}$

Therefore we have by using the geometric series

$$\frac{1}{1+x} = 1 - x + x^2 - x^3 + \cdots$$

Here radius of the circle is unity so circumference of the circle is = 2π .1

We consider the 1/8 part of circumference (by drawing a diagonal in the square) of the circle of unit radius which is arc A_0 L and equivalent to the side of the square.

$$\frac{\text{circumference}}{8} = \frac{2\pi.1}{8} = \lim_{n\to\infty} \sum_{r=1}^{n} \frac{1/n}{1+(\frac{r-1}{n})^2} \quad \text{By expansion}$$

$$= \lim_{n\to\infty}\left(\frac{1}{n} \sum_{r=1}^{n-1} 1 - \frac{1}{n^3}\sum_{r=1}^{n-1} r^2 + \frac{1}{n^5} \sum_{r=1}^{n-1} r^4 - \dots \right)$$

The ancient Kerala mathematicians knew that for any integer $p \geq 0$,

$\frac{1}{n^{p+1}}\sum_{r=1}^{n-1} r^p$ Tends to $\frac{1}{p+1}$ as n tends to ∞ this result was proved in Europe by Roberval in 1634 AD nearly a centuries later than discovery of the Kerala mathematicians.

Using this in above result we have

$\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \dots$. By this series we can get the value of π.

Madhava had even better approximation to π = 3.1415926536 (quoted in Karanapaddhati , IV ,7). But Madhava was not satisfied and he gave another better rational approximation to π = 3.14159265359.

Bhaskara [9] II also gave the value of $\pi = \frac{3927}{1250} = 3.1416$.

Ganesha Daivaghya [10] (Buddhivilasini) suggested that the ratio of the perimeter of the 384 sides polygon inscribed in a circle of diameter 100 would give the approximate value of π as $\frac{3927}{1250}$.

3. The value of π in Jaina Texts

The ratio of the circumference of a circle to its diameter was also known to ancient Jaina mathematicians.

$\pi = \frac{\text{circumference}}{\text{diameter}} = \frac{62832}{20000} = 3.1416,$ This value is correct up to four places of decimals. Apart from this, the two values 3 and $\sqrt{10}$ for π were also known to Jaina scholars. Virasenacharya has quoted from some earlier work that the value of π in the following form :

If Circumference of the circle = C

And its diameter = D

Then $C = 3D + \frac{16D + 16}{113}$ was the formula according to the couplet. Virasenacharya interpreted the above as (app.)

$$C = 3D + \frac{16D}{113} = \frac{355D}{113}$$

Which gives the ratio of C to D which we say $\pi = \frac{355}{113}$

A. N. Singh's [11] remarks, the term 'Sahitam' was used in the sense of addition as well as multiplication i.e. Repeated addition, a number of times in the Vedanga Jyotisha , but it has been used in that double sense by Aryabhata I (c + 499) and succeeding mathematicians. This leads to conclude that the above quotation is from some work written before the fifth century AD, when "Sahitam" was being used in both the senses of multiplication as well as addition.

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It would appear, therefore that the so-called Chinese value of $\pi = \frac{355}{113}$ was also known in India and was perhaps used in India earlier than in China. It might be that the Chinese got this value from India, through Buddhist missionaries or perhaps they found out the value independently. According to him, "Another noteworthy feature in the following quotation is the remark 'finer than the fine'; from this it follows that a 'fine' value of π was already known". This fine value of π may have been $\sqrt{10}$ or $\frac{22}{7}$. In this connection Aryabhat's I value is obvious — the third convergent being a close approximation than the second [12]. In Jaina text the formula for finding the circumference of circle from given diameter is given as

$C = \sqrt{10(\text{diameter})^2}$ or $C = \sqrt{10(d)^2}$,

Where d = diameter, and C is circumference , in which place of π the scholar considered $\sqrt{10}$. This value was also used for many years by mathematicians of other countries.

Here the ratio $\frac{C}{d}$ was used for π and its value $= \sqrt{10}$ was used in many ancient texts of Jaina literature. It was used as a gross value by Brahmgupta (628 AD) in Brahma Sphuta -Siddhanta , Virasena (816 AD) , Mahavira (850 AD) and others [13].
Virasena used π = 3 in Dhavala commentary on the Shatkhandagama[14], (V1.4.8 P.169). The Jainas have approved $\pi = \sqrt{10}$ and this value for π had been adopted from early period in Jaina work [15] .

Examples of this value of π are given in the following ancient Jaina texts.

1. Anuyogadvara Sutra (Chulika Sutra) written by Aryarakshita in (Sutra 164)
2. Bhagavati Sutra of Sudharama Svami in (Sutra 91)
3. Jivajivibhagama Sutra in (Sutra 82, 109)
4. Surya pragyapti , it is one of the Anga of Jaina literature in (Sutra 20 / 132)
5. Jambu dvipa pannatti , it is one of the Anga of Jaina literature in (Sutra 3 /132 and 149)
6. Tattvarthadhigma Sutra Bhashya of Umasvati (III , 11)
7. Jambu dvipa Samasa by Umasvati 1st century
8. Jyotishakarandaka written by Vallabhi Acharya probably of 300 AD (Gatha 185)
9. Tiloyapannatti of Yativrshabha (1 /117)
10. Ganita- Sara - Sangraha of Mahaviracharya (850 A(VII , 60)
11. Tiloyasara of Nemichandra Siddhanta – Chakravarti(Gatha 96)

We find two value 3 and $\sqrt{10}$ for π in Trilokasara [16] .

Other value of this ratio π is $\left(\frac{16}{9}\right)^2$, referred by B. B. Datta [17], M. B. L. Agrawal [18], L C Jain [19] and Takao Hayashi [20] , it is derived from Aryabhata's value [21].

The value of π is approximated to the square root of 10 and calculated correct up to 13 places of decimal in Suryapragyapti an astronomical work of Jaina . Trilokasara is an outstanding work of Prakrita literature. It is mentioned in (Verse 17, p. 18) which means that diameter multiplied by 3 gives circumference.
Therefore C = 3 d , where C is the circumference and d is the diameter implies π = 3. Here 3 is the gross value for π. Nemichandra acharya was aware that the above formula which is an approximation and not the exact one. It is clear from another Gatha which states that the circumference of a circle is three times the diameter and a little more i.e. C = 3d +s , where s is a savishesha. He gave another formula [22] from Trilokasara which state that the square root of ten multiplied by the square of the diameter becomes the circumference. Therefore $C = \sqrt{10d^2}$, it implies the value of $\pi = \sqrt{10}$, which is a better approximation. In Babylonia [23] (C 1700 BC) , Chinese work of Chou Pei [24] and in the Talmud [25] this value is used .

The Value of π in other countries:

The value of π in China, Japan, Arab and other countries was similar to that of Jaina value . The Chinese scholar Chang Heng in 2nd century AD used first time the value of $\pi = \sqrt{10}$. However, how he obtained this value was not known .

The commentary written on text "Nine chapters of arithmetic" of Chiu Chang had given the following result.

$$\left(\frac{\text{circumference of circle}}{\text{perimeter of external square}}\right)^2 = \frac{5}{8}$$

Or $(\text{circumference of circle})^2 = 16 \times 5 \times \frac{(\text{daimeter})^2}{8}$

Or $(\text{Circumference of circle})^2 = 10 \times (\text{daimeter})^2$

In the 9th century Alakhvarijami of West Asia mentioned in Algebra (Arabic language) that the formula for finding the circumference of the circle was

$$(\text{circumference of circle})^2 = 10 \times (\text{daimeter})^2$$

It is based on the translation of Jaina and Vedic texts in 8th century AD . Even in Spain Al- Zarqali in 11th century AD gave different approximations of π in his Arabic texts in which one of the value of π is equal to $\sqrt{10}$. Later on, these texts were translated into Latin language and in one of the book Canonessive Regule Super Tabulas Toletannas the value of π is expressed by the formula.

$$\frac{\text{circumference of circle}}{\sqrt{10}} = \text{diameter}$$

This result is similar to the value obtained by Jaina scholars. Needham has given the formula of Chang Heng that is different and seems to be incorrect. The value $\pi = \sqrt{10}$ was used afterwards in China [26] Imaishin [27] stated that this value came through India via China and he has mentioned the reference of Anuyogadvara Sutra a Jaina text written by Aryarakshita . In the 9th century Alakhvarijami of West Asia mentioned in Algebra (Arabic language) that the formula for finding the circumference of the circle was

$$(\text{circumference of circle})^2 = 10 \times (\text{daimeter})^2$$

Al - Tabari in 11th century wrote Miftah al-Muamlat in Persian language in that book the formula $\sqrt{10(\text{diameter})^2}$ for circumference of circle is mentioned along with several other formulae . In the Book

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Sindh - Hind, it is used frequently. In Hebrew language the writer Abraham Ibn Ezra [28] in 12^{th} century AD the value of $\pi = \sqrt{10}$ is mentioned and stated that it is Indian value. Johannes Dulineris 15^{th} century AD, in his text Canones Tabularum gave a law whose meaning is

$$\frac{(\text{circumference of circle})\ 2}{10} = (\text{diameter})^2$$

Johannes Buteo (1492 AD) in his book Dequadrature Circuli had used the Jaina value of $\pi = \sqrt{10}$, but the cerdit was given to Arabians . Charles Bovillus wrote a famous book on geometry which was published in Latin , French and Dutch languages in 1507 AD , in which (Introducetorium Geometricum) the value of π was taken as $\sqrt{10}$.
Gumnden also mentioned the same formula. Peurbach in his text Tractates Super Properness (Nuremberg, 1541 AD) also quoted the same formula. Regiomontanus considered $\pi = \sqrt{10}$ as approximate value from Arab, as we all know that the science and mathematics of India was first translated into Arabic and then it spread into Europe [29].
This value of $\pi = \sqrt{10}$ was also used in Japan in 17^{th} century, particularly for the formulae for arc, chord and other circular measurement of the circle.
In 18th century AD Chhien Thang was of the opinion that this value of $\pi = \sqrt{10}$ is convenient and near to the correct value

Conclusion:

The contributions of Indian mathematicians during ancient and medieval periods have been noteworthy. We have shown that the Vedic and Jaina scholars had the knowledge of the value of π but in different forms. The Vedic and Jaina philosophical texts present the concept of π. Textual evidences show that the ratio of the circumference to the diameter was known to the ancient mathematicians of India but still we require the world to understand and recognize our contribution. There may be many reasons for it, one reason is writing the mathematical expressions in verse (Gatha) form and second our traditional approach of not giving much importance to scientific and mathematical thoughts. Let us hope that mathematics will be pursed and developed to its pristine glory in India.

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